THE KIRCHHOFF FORMULA

FOR A

SUPERSONICALLY MOVING SURFACE

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ABSTRACT

The Kirchhoff formula for radiation from stationary surfaces first appeared in 1882, and it has since found many applications in wave propagation theory. In 1930, Morgans extended the formula to apply to surfaces moving at speeds below the wave propagation speed; we refer to Morgans' formula as the subsonic formulation. A modern derivation of Morgans' result was published by Farassat and Myers in 1988, and it has now been used extensively in acoustics, particularly for high speed helicopter rotor noise prediction. Under some common conditions in this application, however, the appropriate Kirchhoff surface must be chosen such that portions of it travel at supersonic speed. The available Kirchhoff formula for moving surfaces is not suitable for this situation. In the current paper we derive the Kirchhoff formula applicable to a supersonically moving surface using some results from generalized function theory. The new formula requires knowledge of the same surface data as in the subsonic case. Complications that arise from apparent singularities in the new formulation are discussed briefly in the paper.

1. INTRODUCTION

Since its first publication in 1882, the Kirchhoff formula has played a fundamental role in the study of optical, acoustic and electromagnetic wave propagation.[1-2] The formula provides a representation of solutions to the homogeneous wave equation in regions interior or exterior to a closed surface in terms of data specified on the surface itself. Here we will call this surface the Kirchhoff surface. The original formula applies only to a stationary Kirchhoff surface. In many circumstances, however, it is convenient to have an analogous formula applicable to a moving surface, and such a result was derived by Morgans,[3] who extended the original formula of Kirchhoff to include surfaces moving at speeds below the wave speed in the propagation medium; we refer to Morgans' result as the subsonic Kirchhoff formula. A modern derivation this formula based on generalized function theory and in a form suitable for application was given by the authors in 1988.[4]

In addition to the fact that representation formulas of the Kirchhoff type are of interest from a fundamental analytical standpoint, they have in recent years also assumed importance as computational tools for use in conjunction with CFD calculations. One illustration of the utility of the subsonic Kirchhoff formula is its successful application by Lyrintzis[5] to helicopter noise prediction. Here unsteady aerodynamic calculations were performed in the near field of a helicopter rotor in a reference frame fixed to the rotating blades (see fig. 1). These then provided data on a blade-fixed Kirchhoff surface like that indicated in fig. 1 for subsequent application of the subsonic Kirchhoff formula to determine the noise radiated by the blade. This particular application, however, is limited by the fact that under some commonly occurring conditions the shock system associated with a high speed helicopter rotor can extend well beyond the tip region, a phenomenon that has been called delocalization.[6] To apply a (linear) Kirchhoff formula in this case requires that the entire shock system be included inside the Kirchhoff surface. Because part of the surface thus travels at supersonic speed, the available Kirchhoff formula for moving surfaces is not suitable.

The primary difficulty with the subsonic formula is the appearance of the Doppler factor $1-M_r$ in the denominator of the integrands, where M_r is the component of the surface Mach number in the radiation direction. For supersonic surfaces, there exist directions in which $M_r=1$, and thus singularities appear in the subsonic Kirchhoff formula. Here we derive a new Kirchhoff formula that does not contain the Doppler singularity. The new formula is actually valid for all surface speeds, but, because it is somewhat complex to code for computer applications, its use is not recommended for Kirchhoff surfaces, or portions or Kirchhoff surfaces, moving subsonically. In this paper, therefore, we refer to the new formula as the supersonic Kirchhoff formula. The analysis is carried out with a view toward practical numerical implementation. Thus, the new formula is derived for an open surface panel considered as a portion of the closed Kirchhoff surface. This allows its use to be restricted to just those parts of the Kirchhoff surface that are actually moving at supersonic speed.

In the next section, the inhomogeneous source terms of the wave equation leading to the Kirchhoff formula are derived. In the following section, the solution of this wave equation is obtained which is valid for supersonically moving surfaces. This solution is the desired Kirchhoff formula. In the subsequent section the singularities associated with the new formula are briefly discussed. The main complications in the supersonic formulation are the existence of multiple emission times and the appearance of small or vanishing denominators in the algebraic expressions. We indicate how these complications can be overcome in applications.

2. THE INHOMOGENEOUS WAVE EQUATION

Our approach to deriving the supersonic Kirchhoff equation is similar to that of reference 4. The primary reference for the mathematics used in the derivation is a recent publication by the first author.[7] Let us assume that the

moving Kirchhoff surface is defined by $f(\vec{x},t)=0$ with $f>0$ in the exterior of the surface and $f<0$ in the interior. This function is selected in such a way that $|\nabla f|=1$ over the surface, which can always be done.[7] We are interested in radiation into the region exterior to the surface $f=0$. Let $\Phi(\vec{x},t)$ satisfy the homogeneous equation in this region:

$$\Box^2 \Phi = \frac{1}{c^2}\frac{\partial^2 \Phi}{\partial t^2} - \nabla^2 \Phi = 0 \qquad (1)$$

in which c is the speed of sound. To find the inhomogeneous source terms of the wave equation leading to the Kirchhoff formula, we extend Φ to the entire three-dimensional space as follows:

$$\tilde{\Phi}(\vec{x},t)=\begin{cases}\Phi(\vec{x},t) & f>0 \\ 0 & f<0\end{cases} \qquad (2)$$

We now find $\bar{\Box}^2\tilde{\Phi}$ where $\bar{\Box}^2$ stands for the wave operator with generalized derivatives.

Using the rules of generalized differentiation,[7-10] we obtain

$$\frac{1}{c}\frac{\bar{\partial}\tilde{\Phi}}{\partial t}=\frac{1}{c}\frac{\partial\tilde{\Phi}}{\partial t}-M_n\Phi\,\delta(f) \qquad (3\text{-}a)$$

$$\frac{1}{c^2}\frac{\bar{\partial}^2\tilde{\Phi}}{\partial t^2}=\frac{1}{c^2}\frac{\partial^2\tilde{\Phi}}{\partial t^2}-\frac{1}{c}M_n\frac{\partial\Phi}{\partial t}\delta(f)-\frac{1}{c}\frac{\partial}{\partial t}[M_n\Phi\,\delta(f)] \qquad (3\text{-}b)$$

In eqs. (3) the bar over a derivative stands for generalized differentiation, and $M_n=v_n/c$ is the local normal Mach number of the surface $f=0$. The Dirac delta function with support on $f=0$ is denoted $\delta(f)$. The generalized Laplacian of $\tilde{\Phi}$ is similarly found as follows:

$$\bar{\nabla}\,\tilde{\Phi}=\nabla\tilde{\Phi}+\Phi\,\vec{n}\,\delta(f) \qquad (4\text{-}a)$$
$$\bar{\nabla}^2\tilde{\Phi}=\nabla^2\tilde{\Phi}+\Phi_n\delta(f)+\nabla\cdot[\Phi\vec{n}\,\delta(f)] \qquad (4\text{-}b)$$

Here $\vec{n}=\nabla f$ is the local unit outward normal and $\Phi_n=\vec{n}\cdot\nabla\Phi$ on $f=0$. From eqs. (3) and (4), and the fact that $\Box^2\tilde{\Phi}=0$, we find

$$\bar{\Box}^2\tilde{\Phi}=\frac{1}{c^2}\frac{\bar{\partial}^2\tilde{\Phi}}{\partial t^2}-\bar{\nabla}^2\tilde{\Phi}$$

$$\qquad (5)$$

$$=-\left(\frac{M_n}{c}\frac{\partial\Phi}{\partial t}+\Phi_n\right)\delta(f)-\frac{1}{c}\frac{\partial}{\partial t}[M_n\Phi\,\delta(f)]-\nabla\cdot[\Phi\vec{n}\,\delta(f)]$$

The last two terms of this equation need further manipulations to make them suitable for the derivation of the supersonic Kirchhoff formula. We perform these manipulations below.

In applications, the Kirchhoff surface is first divided into panels (Fig. 2-a). Some of these panels travel at supersonic speed, so that the subsonic Kirchhoff formula is not suitable. We therefore, derive the new formula for a single panel. Let

us assume that a panel on $f=0$ is defined by the equation of its edge curve, $\tilde{f}=0$. We assume that $\tilde{f}>0$ on the panel. We can always define $\tilde{f}$ in such a way that $\nabla\tilde{f}=\vec{v}$ where $\vec{v}$ is the local unit inward geodesic normal at the edge of the panel. The geodesic normal is tangent to the surface $f=0$ and is normal to the edge of the panel $f=\tilde{f}=0$ (see Fig. 2-b). For the open surface at this panel, the source terms of the wave equation in eq. (5) must be modified by the Heaviside function $H(\tilde{f})$ as follows:

$$\bar{\Box}^2\tilde{\Phi}=-\left(\frac{M_n}{c}\frac{\partial\Phi}{\partial t}+\Phi_n\right)H(\tilde{f})\,\delta(f)$$
$$\qquad (6)$$
$$-\frac{1}{c}\frac{\partial}{\partial t}[M_n\Phi H(\tilde{f})\,\delta(f)]-\nabla\cdot[\Phi\vec{n}H(\tilde{f})\,\delta(f)]$$

Now we carry out the time differentiation and divergence in the last two terms of eq. (6). We have

$$\frac{1}{c}\frac{\partial}{\partial t}[M_n\Phi H(\tilde{f})\,\delta(f)]=\frac{1}{c}\frac{\partial}{\partial t}(\underset{\sim}{M}_n\Phi)H(\tilde{f})\,\delta(f)$$
$$\qquad (7)$$
$$-M_nM_v\Phi\,\delta(f)\,\delta(\tilde{f})-M_n^2\underset{\sim}{\Phi}H(\tilde{f})\,\delta'(f)$$

where a tilde under a function stands for the restriction of the function to the surface $f=0$.[7] The symbol $M_v=v_v/c$ is the local Mach number of the edge $f=\tilde{f}=0$ in the direction of the geodesic normal. Similarly, the last term in eq. (6) can be written as

$$\nabla\cdot[\Phi\vec{n}H(\tilde{f})\,\delta(f)]=-2H_f\Phi H(\tilde{f})\,\delta(f)+\underset{\sim}{\Phi}H(\tilde{f})\,\delta'(f) \qquad (8)$$

where H_f is the local mean curvature of the surface $f=0$. After substituting eqs. (7) and (8) into eq. (6) and collecting terms, we get

$$\bar{\Box}^2\tilde{\Phi}=-\left[\Phi_n+\frac{M_n}{c}\frac{\partial\Phi}{\partial t}+\frac{1}{c}\frac{\partial}{\partial t}(\underset{\sim}{M}_n\Phi)-2H_f\Phi\right]H(\tilde{f})\,\delta(f)$$
$$\qquad (9)$$
$$-(1-\underset{\sim}{M}_n^2)\underset{\sim}{\Phi}H(\tilde{f})\,\delta'(f)+M_nM_v\Phi\,\delta(f)\,\delta(\tilde{f})$$

This is the desired form of the inhomogeneous source terms of the wave equation. In the next section, we give the solution of eq. (9) which is the supersonic Kirchhoff formula.

3. THE SUPERSONIC KIRCHHOFF FORMULA

Equation (9) is now written as

$$\bar{\Box}^2\tilde{\Phi}=q_1(\vec{x},\tau)H(\tilde{f})\,\delta(f)+q_2(\vec{x},\tau)H(\tilde{f})\,\delta'(f)$$
$$\qquad (10)$$
$$+q_3(\vec{x},\tau)\,\delta(\tilde{f})\,\delta(f)$$

Note that q_2 is restricted to $f=0$ and hence is written as $\underset{\sim}{q}_2$.

Next we write $\tilde{\Phi}$ as the sum of three functions

$$\tilde{\Phi} = \Phi_1 + \Phi_2 + \Phi_3 \qquad (11)$$

where these functions are the solutions of the wave equations

$$\vec{\Box}^2 \Phi_1 = q_1(\vec{x},t)H(\tilde{f})\delta(f) \qquad (12\text{-a})$$

$$\vec{\Box}^2 \Phi_2 = q_2(\vec{x},t)H(\tilde{f})\delta'(f) \qquad (12\text{-b})$$

$$\vec{\Box}^2 \Phi_3 = q_3(\vec{x},t)\delta(\tilde{f})\delta(f) \qquad (12\text{-c})$$

The solutions of these equations are given fully in reference 7 (see solutions of eqs. 4.23-b, e, f in Ref. 7). Here we utilize these results directly and refer readers to the original source.

The solution of eq. (12-a) is given by eq. (4.34) of reference 7 as follows:

$$4\pi\,\Phi_1(\vec{x},t) = \int_{\substack{F=0 \\ \tilde{F}>0}} \frac{Q_1}{r\Lambda}\,d\Sigma \qquad (13)$$

where $Q_1 = [q_1(\vec{y},\tau)]_{ret} = q_1(\vec{y},t-r/c)$, $r = |\vec{x}-\vec{y}|$,
$F(\vec{y};\vec{x},t) = [f(\vec{y},\tau)]_{ret}$, $\tilde{F}(\vec{y};\vec{x},t) = [\tilde{f}(\vec{y},\tau)]_{ret}$ and

$$\Lambda^2 = 1 + M_n^2 - 2M_n\cos\theta \qquad (14)$$

Here M_n is local normal Mach number on the panel and $\cos\theta = \vec{n}\cdot\hat{r}$, where $\hat{r} = (\vec{x}-\vec{y})/r$ is the unit radiation vector. In eq. (13) $d\Sigma$ is the element of surface area of the surface $F=0, \tilde{F}>0$. This is the surface formed by the curves of intersection of the collapsing sphere $r = c(t-\tau)$, $(\vec{x},t)$ fixed, with the panel in motion as the source time τ increases from $-\infty$ to t. Figure 3 illustrates the construction of the Σ-surface for a panel.

The solution of eq. (12-b) is given by eq. (4.42) of reference 7 in the form

$$4\pi\,\Phi_2(\vec{x},t) = \int_{\substack{F=0 \\ \tilde{F}>0}} \left[-\frac{1}{\Lambda}\frac{\partial}{\partial N}\left(\frac{Q_2}{r\Lambda}\right) + \frac{2H_F Q_2}{r\Lambda^2} \right]d\Sigma$$

$$- \int_{\substack{F=0 \\ \tilde{F}=0}} \frac{Q_2\cot\theta'}{r\Lambda^2}\,dL \qquad (15)$$

where $Q_2 = [q_2(\vec{y},\tau)]_{ret}$ and we have introduced the following symbols:

$$\frac{\partial}{\partial N} = \vec{N}\cdot\nabla\,, \qquad \vec{N} = \frac{\vec{n} - M_n\vec{\hat{r}}}{\Lambda} = \frac{\nabla F}{|\nabla F|} \qquad (16\text{-a,b})$$

$$\cos\theta' = \vec{N}\cdot\vec{N}'\,, \qquad \vec{N}' = \frac{\vec{v} - M_v\vec{\hat{r}}}{\tilde{\Lambda}} = \frac{\nabla\tilde{F}}{|\nabla\tilde{F}|} \qquad (16\text{-c,d})$$

$$\tilde{\Lambda}^2 = 1 + M_v^2 - 2M_v\cos\tilde{\theta} \qquad (16\text{-e})$$

$$\cos\tilde{\theta} = \vec{v}\cdot\vec{\hat{r}} \qquad (16\text{-f})$$

In eq. (15) we have also denoted the mean curvature on the Σ-surface by H_F, and dL is the element of the length of the edge of the Σ-surface. Below we will see why we must retain the restriction sign ($\sim$) in Q_2 in the integrand of the surface integral.

The solution of eq. (12-c) is given by eq. (4.49) of reference 7 as follows:

$$4\pi\,\Phi_3(\vec{x},t) = \int_{\substack{F=0 \\ \tilde{F}=0}} \frac{Q_3}{r\Lambda_0}\,dL \qquad (17)$$

where $Q_3 = [q_3(\vec{y},\tau)]_{ret}$ and

$$\Lambda_0 = |\nabla F \times \nabla\tilde{F}| = \Lambda\tilde{\Lambda}\sin\theta' \qquad (18)$$

After combining the eqs. (13), (15) and (17) we get

$$4\pi\tilde{\Phi}(\vec{x},t) = \int_{\substack{F=0 \\ \tilde{F}>0}} \frac{1}{r\Lambda}\left[Q_1 + \frac{2H_F}{\Lambda}Q_2 + \frac{\vec{N}\cdot\nabla\Lambda}{\Lambda^2}Q_2 - \frac{\vec{N}\cdot\nabla Q_2}{\Lambda} \right]d\Sigma$$

$$+ \int_{\substack{F=0 \\ \tilde{F}>0}} \frac{\vec{N}\cdot\nabla r}{r^2\Lambda^2}Q_2\,d\Sigma + \int_{\substack{F=0 \\ \tilde{F}=0}} \frac{1}{r\Lambda_0}\left[Q_3 - \frac{\tilde{\Lambda}\cos\theta'}{\Lambda}Q_2 \right]dL \qquad (19)$$

In eq. (19) we have explicitly performed the derivative $\partial/\partial N$ in eq. (15) and separated the near and far field surface integrals. We note that

$$\vec{N}\cdot\nabla Q_2 = [\vec{N}\cdot\nabla q_2]_{ret} - \frac{1}{c}[\vec{N}\cdot\nabla r\,\dot{q}_2]_{ret} \qquad (20)$$

where the symbol $\dot{q}_2$ is used for $\partial q_2(\vec{y},\tau)/\partial\tau$. From eq. (16-b), we have

$$\vec{N} = \frac{1}{\Lambda}\left[\vec{n} - M_n(\vec{n}\cos\theta + \vec{t}_1\sin\theta)\right] = \frac{1}{\Lambda}(1 - M_n\cos\theta)\vec{n}$$

$$+ \frac{M_n\sin\theta}{\Lambda}\vec{t}_1 \qquad (21)$$

in which $\vec{t}_1$ is the unit vector along the projection of $\vec{\hat{r}}$ on the local tangent plane of f=0. Since $\partial q_2/\partial n = 0$ (this a property of the restriction[7]), we get

$$\vec{N} \cdot \nabla q_2 = \frac{M_n \sin\theta}{\Lambda} \frac{\partial q_2(\vec{y}, \tau)}{\partial \sigma_1} \qquad (22)$$

where $\partial/\partial\sigma_1$ is the directional derivative in the direction of $\vec{t}_1$ (keeping τ fixed). We also have

$$\vec{N} \cdot \nabla r = \frac{M_n - \cos\theta}{\Lambda} \qquad (23)$$

which, when substituted in eq. (20), gives

$$\vec{N} \cdot \nabla Q_2 = \frac{M_n \sin\theta}{\Lambda} \frac{\partial Q_2}{\partial \sigma_1} + \frac{\cos\theta - M_n}{c\Lambda} \dot{Q}_2 \qquad (24)$$

In eq. (24), $\dot{Q}_2$ stands for $(\partial q_2/\partial\tau)_{ret}$ and the directional derivative again is calculated keeping τ fixed. Finally, after using eqs. (23) and (24) in eq. (19), we get

$$4\pi\, \bar{\Phi}(\vec{x}, t) = \int\limits_{\substack{F=0 \\ \bar{F}>0}} \frac{1}{r\Lambda} \left[Q_1 + \left(\frac{2H_F}{\Lambda} + \frac{\vec{N} \cdot \nabla\Lambda}{\Lambda^2} \right) Q_2 \right.$$
$$\left. - \frac{M_n \sin\theta}{\Lambda^2} \frac{\partial Q_2}{\partial \sigma_1} - \frac{\cos\theta - M_n}{c\Lambda^2} \dot{Q}_2 \right] d\Sigma \qquad (25)$$
$$- \int\limits_{\substack{F=0 \\ \bar{F}>0}} \frac{\cos\theta - M_n}{r^2\Lambda^3} Q_2\, d\Sigma + \int\limits_{\substack{F=0 \\ \bar{F}=0}} \frac{1}{r\Lambda_0} \left[Q_3 - \frac{\bar{\Lambda}\cos\theta'}{\Lambda} Q_2 \right] dL$$

Equation (25) is the supersonic Kirchhoff formula and is the main result of this paper. We will discuss the above result further below.

First, however, we remark on the significance of utilizing the restriction of the function q_2 to the surface $f=0$ in the derivation of eq. (25). The key property of the restriction is that $\partial q_2/\partial n = 0$, which allows the derivative in the direction $\vec{N}$ in eq. (15-b) to be evaluated very simply (see eq. (22)). If we had not introduced the restriction, the ultimate result would still be eq. (25), but it would contain a number of extra terms arising from unnecessary differentiation of q_2 in the direction $\vec{n}$. The extra terms, of course, cancel one another, but is difficult to recognize the cancellation in the form in which the terms arise in eq. (25). Using q_2 leads directly to the above result which the authors believe is the simplest possible expression of the supersonic Kirchhoff formula.

One important fact that we point out here is that eq. (25) is valid for a Kirchhoff surface in arbitrary motion, including motion at subsonic speed. However, we do not recommend its use for subsonic speeds because the formula presented by the authors in reference 4 is much more efficiently applied for computation.

4. ANALYSIS OF THE MAIN RESULT

We first check to see if we recover the classical

Kirchhoff formula from eq. (25) when the surface is stationary. We assume a closed surface described by $f(\vec{y}) = 0$. In this case there is no line integral and we have

$$F(\vec{y}; \vec{x}, t) = f(\vec{y}) = 0 \qquad (26\text{-}a)$$

$$M_n = 0 , \qquad \Lambda = 1 \qquad (26\text{-}b,c)$$

$$H_F = H_f , \qquad q_1 = -\Phi_n - 2H_f \Phi \qquad (26\text{-}d,e)$$

$$q_2 = \Phi , \qquad q_3 = 0 , \qquad d\Sigma = dS \qquad (26\text{-}f,g,h)$$

where dS is the element of the surface area of $f=0$. Substitution of these results in eq. (25), yields

$$4\pi\, \bar{\Phi}(\vec{x}, t) = \int\limits_{f=0} \frac{1}{r} \left[c^{-1}\cos\theta\, \dot{\Phi} - \Phi_n \right]_{ret} dS + \int\limits_{f=0} \frac{\Phi\cos\theta}{r^2} dS$$

$$(27)$$

This is the classical Kirchhoff formula.[1,2] Thus this aspect of consistency of our main result has been established. It is evident that eq. (25) depends on Φ, Φ_n and $\dot{\Phi}$ on the Kirchhoff surface $f=0$. This is, of course, expected. Note that $\partial Q_2/\partial\sigma_1$ involves $\partial\Phi/\partial\sigma_1$, which is obtained from knowledge of Φ on $f=0$.

We will now discuss the problem of singularities of eq. (25). The quantity Λ appears in every term of the denominator and produces a singularity when $\Lambda=0$. The singularity $\Lambda_0 = 0$ appears in the line integral of eq. (25) if the conditions discussed in reference 11 are satisfied. When $\Lambda=0$, the collapsing sphere is tangent to the Kirchhoff surface $f=0$ at a point where $M_n=1$. The Σ surface in the vicinity of this point is very complicated with a possible mean curvature singularity. This case requires further analysis which we will not pursue at present. For now we will indicate a practical method to get around this problem. Assume that the Kirchhoff surface is chosen to have a shape like that of a biconvex airfoil such that there are no points on the surface at which $M_n=1$. Then we only have to be concerned with the singularities of the line integral in eq. (25) on the leading and trailing edges of the Kirchhoff surface. However, the integrand of the line integral of eq. (25) is precisely eq. (17-c) of reference 11 with $\rho'c^2$ replaced by Φ. An analysis identical to that of reference 11 shows that the singularity of the line integral in eq. (25) is integrable so long as Φ is continuous in the vicinity of the singular point. But this condition is always satisfied in practice so that there will be no numerical problems in using eq. (25) for the proposed Kirchhoff surface.

5. CONCLUDING REMARKS

In this paper we have derived a Kirchhoff formula that describes solutions of the homogeneous wave equation exterior to a surface in arbitrary motion. The formula is specifically designed for practical computation of radiation from surfaces in supersonic motion. Because surfaces moving at subsonic speeds are more efficiently treated using an earlier result, the new formula is derived for an open surface; it is recommended that it be used only on surface

panels that are actually moving supersonically. The new formula is expressed in a relatively simple form in terms of surface data and of easily calculated geometric and kinematic properties of the moving Kirchhoff surface. It is complicated somewhat by the existence of singularities that occur under certain conditions in supersonic motion, but these singularities are shown to present no difficulties in numerical implementation of the formula.

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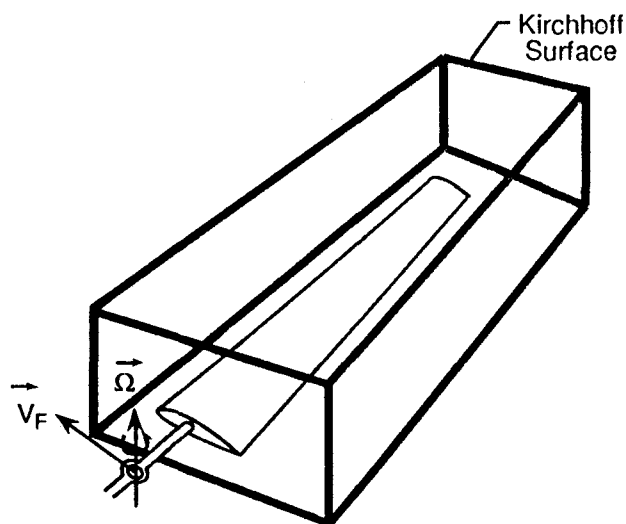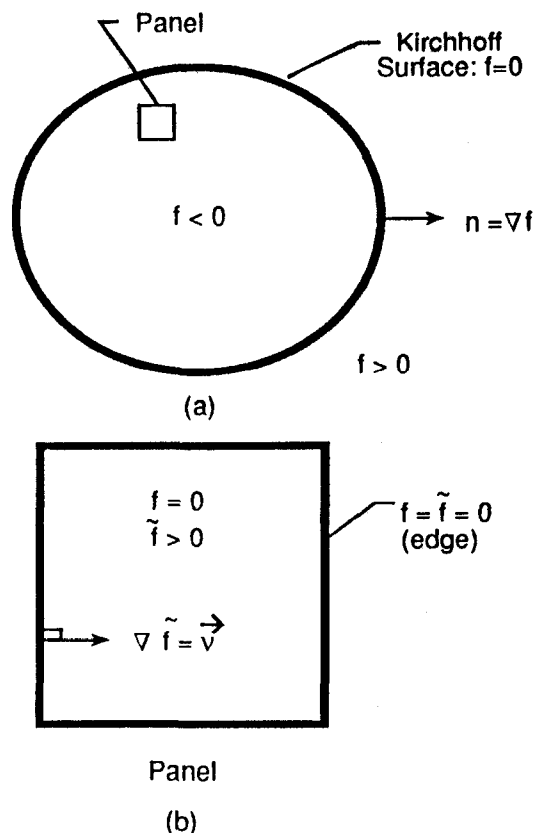
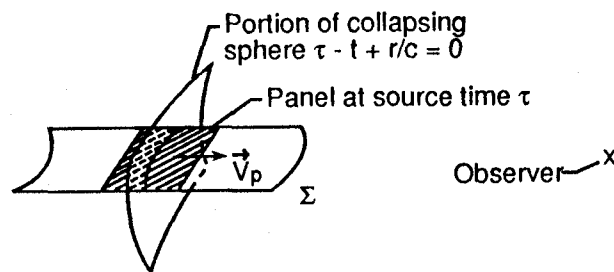Fig. 2   Open panel on Kirchhoff surface.

Fig. 1   Rotating Kirchhoff surface for helicopter blade.

Fig. 3   Construction of Σ-surface for panel.